Comparison of Reliability Analysis Between Regression Kriging and Sensitivity Assisted MCS Methods for Reliability-based Optimal Design of Electromagnetic Devices

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For the reliability analysis of electromagnetic devices under uncertainty, numerous function calls are usually required during the performance analysis process of reliability-based design optimization, which actually causes a heavy numerical burden and low-efficiency. The main contribution of this paper is to propose an efficient reliability analysis method that combines Monte Carlo simulation (MCS) with regression Kriging surrogate model. Numerical experiments on analytic and benchmark problems are employed to investigate the performance of proposed reliability analysis method. It is also compared with the existing methods such as reliability index approach, the sensitivity-assisted MCS methods, and the conventional MCS method.

***Index Terms*—Reliability analysis, reliability-based design optimization, regression Kriging, Monte Carlo simulation.**

I. Introduction

In order to find the reliable design against uncertainty, recently, the reliability analysis and reliability-based design optimization (RBDO) are introduced into the field of electrical engineering [1]. The RBDO, which integrates reliability analysis and deterministic optimization approaches, can search for a reliable optimum under uncertainty. The accuracy and efficiency of reliability calculation method directly affect the computational cost of the RBDO algorithm, and even decide whether the RBDO can find a true reliable solution or not.

The Monte Carlo simulation (MCS), is known as an accurate method for reliability analysis when the number of trials is large enough (normally more than one million for low-dimensional problems) but suffers from a heavy numerical burden simultaneously especially for finite element models. To mitigate expensive calculation, the sensitivity-assisted MCS method is proposed in [2], where the first-order and the second-order sensitivity analyses are respectively introduced and a higher efficiency is also achieved together with a better accuracy. However, sensitivity analysis of practical engineering problems requires stronger theoretical background on finite element analysis, which is not easy to be popularized. The surrogate modeling techniques such as Kriging and radial basis function based response surface method, have been successfully utilized to assist optimum searching especially for large-scale or complex electromagnetic device problems [3]-[4], which can sharply reduce the computing time.

In this paper, considering the accuracy of MCS method and the efficiency of surrogate modeling technique, a regression Kriging-assisted MCS method is proposed to implement reliability calculation in the reliability-based optimal design. For a validation and comparison of the performance, the proposed method, reliability index approach (RIA), first-order sensitivity-assisted MCS (FSA-MCS) and second-order sensitivity-assisted MCS (SSA-MCS) methods are applied to analytic test functions of different nonlinearity and benchmark TEAM problem 22.

II. A novel Regression Kriging-Assisted Monte Carlo Simulation Method for Reliability Calculation

A. Reliability Analysis in RBDO

A typical RBDO algorithm treats reliability as probabilistic constraints [5]. Reliability (R) of a specified design $\mathbf{x}$ ($\mathbf{x}\in R^n$) under uncertainty is defined as the probability of keeping $\mathbf{x}$ in the feasible region of a constraint $g(\mathbf{x})\leq 0$ as

$$R = P[g(\mathbf{x}) \leq 0]\,. \tag{1}$$

The multi-dimensional integral in (1) makes direct calculation impractical especially for complicated engineering cases.

B. Regression Kriging- Assisted Monte Carlo Simulation

In the proposed method, firstly the surrogate model is built by regression Kriging (RK) as follows:

Step 1. Initialize sampling and testing points: generate sampling points and prepare testing points.

Step 2. Construct regression Kriging model: calculate constraint function values for all sampling points by finite element analysis (FEA).

Based on the known data, formulate RK model as:

$$\mathrm{z}^*(\mathbf{x}_0) = \mathbf{q}_0^T \cdot \boldsymbol{\beta}^* + \boldsymbol{\lambda}_0^T \cdot (\mathbf{z} - \mathbf{q} \cdot \boldsymbol{\beta}^*) \tag{2}$$

where $\mathrm{z}^*(\mathbf{x}_0)$ is the response value at the predicted point $\mathbf{x}_0$ and $\mathbf{z}$ is constraint function value at samples; $\mathbf{q}_0$ and $\mathbf{q}$ are predictors of $\mathbf{x}_0$ and samples, respectively; $\boldsymbol{\lambda}_0$ is Kriging weights and $\boldsymbol{\beta}^*$ is estimated regression coefficients [6].

Calculate the estimated fitting error for all test points, check stop criteria and terminate.

Step 3. Adaptive sampling insertion: insert new sampling points based on the predefined fitting error and go to step 2.

Then, in the RBDO, reliability analysis by the regression Kriging-assisted MCS (RK-MCS) is implemented as:

Step 1. Generate N pseudo-random test points $\boldsymbol{\xi}_i$ in uncertain set based on statistical distribution of random variables.

Step 2. Calculate the constraint value $g(\mathbf{x})$ by the RK model and check if it satisfies $g(\mathbf{x})\leq 0$ or not for each test point $\boldsymbol{\xi}_i$.

Step 3. Evaluate the reliability of design **x** by:

$$R(g(\mathbf{x}) \le 0) = 1/N \cdot \sum_{i=1}^{N} I[g(\xi_i)] \quad (3)$$

and the indicator function $I[\cdot]$ is defined as follows:

$$I[g(\xi_i)] = \begin{cases} 0, & \text{if } g(\xi_i) > 0 \\ 1, & \text{if } g(\xi_i) \le 0 \end{cases} \quad (4)$$

which means only the test points locating in the feasible region contribute to reliability calculation.

III. Numerical Applications

A. Numerical Results of Analytic Test Problems

Two test functions defined in [2] are adopted for performance investigation of different reliability calculation methods: RIA, FSA-MCS, SSA-MCS, RK-MCS, and MCS. For test problem 1, the reliabilities calculated by SSA-MCS and RK-MCS show better consistence with that from MCS as shown in Table I.

For a further performance investigation, test problem 2 with stronger nonlinearity is exploited and the results of different methods are compared in Table II. For different test designs, the RK-MCS with 50 samples and SSA-MCS show better accuracy, which outperform their counterparts RIA, FSA-MCS. Furthermore, once one accurate surrogate model is constructed, the accuracy of RK-MCS for reliability calculation will not be influenced by the standard deviations.

TABLE I Reliability Calculation Results Of Test Problem 1

Designs	Methods	Different standard deviations (**σ**)				
		0.1	0.2	0.3	0.5	0.8
A (3.160, 2.150)	RIA	0.8138	0.6722	0.6169	0.5708	0.5443
	FSA-MCS	0.8327	0.6834	0.6246	0.5752	0.5476
	SSA-MCS	0.8357	0.6776	0.6119	0.5504	0.5074
	RK-MCS [a]	0.8360	0.6778	0.6119	0.5503	0.5060
	MCS	0.8355	0.6779	0.6118	0.5503	0.5062
B (3.297, 2.905)	RIA	1.0000	0.9988	0.9782	0.8870	0.7754
	FSA-MCS	1.0000	0.9999	0.9808	0.8791	0.7525
	SSA-MCS	1.0000	1.0000	0.9939	0.9051	0.7699
	RK-MCS	1.0000	1.0000	0.9935	0.9052	0.7724
	MCS	1.0000	1.0000	0.9934	0.9052	0.7714

[a] The number of samples in RK is 30, other parameters are same as in [2].

TABLE II Reliability Calculation Results of Test Problem 2

σ	Methods	Design A		Design B		Design C	
		R	δ_R (%)	R	δ_R (%)	R	δ_R (%)
0.1	RIA	0.9612	1.908	0.8817	2.682	0.9757	2.205
	FSA-MCS	0.9914	1.174	0.8863	2.174	0.9999	0.221
	SSA-MCS	0.9803	0.316	0.9051	0.099	0.9944	0.331
	RK-MCS	0.9796	**0.031**	0.9052	**0.088**	0.9977	**0**
	MCS	0.9799	—	0.9060	—	0.9977	—
0.2	RIA	0.8112	5.942	0.7230	3.484	0.8380	6.116
	FSA-MCS	0.8662	13.125	0.7223	3.578	0.9284	17.564
	SSA-MCS	0.7632	0.327	0.7471	0.267	0.7953	0.709
	RK-MCS	0.7645	**0.157**	0.7476	**0.200**	0.7902	**0.063**
	MCS	0.7657	—	0.7491	—	0.7897	—
0.3	RIA	0.7218	15.952	0.6534	4.905	0.7446	27.282
	FSA-MCS	0.7678	23.341	0.6525	5.036	0.8299	41.863
	SSA-MCS	0.5984	3.872	0.6827	0.640	0.5994	2.462
	RK-MCS	0.6203	**0.353**	0.6857	**0.204**	0.5865	**0.256**
	MCS	0.6225	—	0.6871	—	0.5850	—

B. Reliability Analysis of TEAM Problem 22

In the design of superconducting magnetic energy storage system, the quenching condition approximated as (5) should be considered.

$$g_i(\mathbf{x}) = |J_i| + 6.4 \cdot |B_{m,i}| - 54.0 \le 0,\ i = 1, 2 \quad (5)$$

where J_i and $B_{m,i}$ (i=1, 2) are the current density and the maximum magnetic flux density respectively. For the three-parameter optimization problem, geometric parameters $\mathbf{x}=[R_2, H_2, D_2]$ are treated as uncertain variables with uncertainty of $\boldsymbol{\sigma}=[15.3,\ 10,\ 10]^T$ mm. Reliability calculation results of five optimal designs chosen from published papers are shown in Table III.

Considering the high computational cost, the number of trials in the MCS based methods is set 10,000. Therefore, in Table III, only the first three decimal values of reliability calculated by MCS methods are accurate enough. Obviously results of RK-MCS with 125 samples and SSA-MCS match well with that of the MCS method. Just considering reliability analysis of a specified design, the computational cost of the RK-MCS (125 times FEA) may be a little higher than the SSA-MCS (around 50~80 times equation solving, depending on nodal meshes and the size of random geometric variables). However, the 125 times FEA in the RK-MCS are just used for surrogate modeling; during the reliability-based optimization process, no more FEA is required while the sensitivity-assisted MCS methods need different times FEA for sensitivity analysis during each iterative process.

From the aspect of application in RBDO, the RK-MCS will be a better choice considering efficiency and accuracy. Besides, the RK-MCS method is more convenient to utilize.

TABLE III Reliability Calculation Results of Team 22

No.	Optimal design **x** [m]			Reliability of $g_2(\mathbf{x})$ [a]			
	R_2	$H_2/2$	D_2	FAS-MCS	SSA-MCS	RK-MCS	MCS
1	3.0800	0.2390	0.3940	0.9805	0.9807	0.9807	0.9807
2	3.0500	0.2460	0.4000	0.7200	0.7232	0.7231	0.7231
3	2.6602	0.5574	0.2218	0.9500	0.9512	0.9522	0.9521
4	3.0988	0.2644	0.3903	0.6679	0.6708	0.6710	0.6716
5	3.0197	0.3081	0.3496	0.5158	0.5215	0.5211	0.5210

[a] Reliability of constraint $g_1(\mathbf{x})$ for all cases is 1.0.

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